THERMAL-LAYER EFFECT IN A MAGNETICALLY ACTIVE PLASMA

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For the first time computer simulation theoretically demonstrates the feasibility of forming a large-scale continuously growing precursor when a shock wave interacts with an extended density perturbation ("thermal-layer" effect) in the case of magnetogasdynamic flows. A criterion is obtained for the formation of an unsteady precursor. The flow pattern in a field parallel to the line of intersection of the shock wavefront and the precursor is shown to be analogous to that from ordinary gasdynamics in many ways.

The thermal-layer effect was detected a rather long time ago (mid-1950's) [1-3] but only in recent years has it been studied in detail theoretically and experimentally [4-7]. The essence of this effect is as follows. When a shock wave interacts with a thin layer or thin channel of reduced density under certain conditions a major global rearrangement ensues, i.e., a precursor is formed with an intensive vortex flow inside it. The sizes of the precursor and the vortex increase continuously with time, far exceeding the thickness of the perturbing layer, and finally this thickness ceases to be of any importance. Since a layer of reduced density is formed fairly frequently as a result of rather weak heating of the gas, it is usually called a thermal layer (hence the name of the effect), although such a layer may also be very hot, i.e., its temperature may greatly exceed the temperature of the unperturbed gas and, conversely, may be produced without any heating at all, e.g., when a "lighter" gas (with a low atomic or molecular weight) is injected.

The condition of Taganov [1] for the formation and unbounded development of a precursor is that the pressure behind the main (unperturbed) shock wave be higher than the gas stagnation pressure in the thermal layer (in the coordinate system bound to the front). Let us note that for strong shock waves the criterion of [1] is satisfied if the density in the layer is only roughly 10% lower than in the unperturbed region.

A thermal layer (TL) can arise spontaneously, e.g., because radiation interacts with the front of a shock wave on the surface along which it propagates, but it can also be produced deliberately, e.g., by heating that surface with a pulse of electric current or, let us say, as a result of electrical or optical breakdown. The TL effect can be used for a variety of purposes, e.g., to induce cumulative processes by the collision of rapidly growing precursor ahead of shock waves propagating toward each other or converging. As mentioned in [8] this effect apparently is responsible for the formation of a quasispherical configuration at the plasma focus, where a thermal layer is produced by the interaction of the radiation with the front of a converging cylindrical wave at the end surface of a cylindrical chamber. A converging plasma sheath was initiated by a powerful electrical discharge in this case. The plasma radiation fluxes in such a Z pinch and the parameters of the heated layer formed were estimated in [9]. Those estimates showed that the thickness of the gas layer heated to a temperature of the order of 10^3 K is small, a mere 10^{-2} cm or thereabouts. On this basis Gerusov and Imshennik [9] concluded that such a mechanism of the "runaway" of a wave along the wall is ineffective. As follows from analysis [1-7], however, in the case of ordinary gasdynamic flows the small thickness of the thermal layer generally speaking is no obstacle to the realization of the TL effect, the formation of a growing precursor and a corresponding distortion of the shape of the converging wave ("runaway" along the wall in the terminology of [8]). Naturally, this conclusion also holds when a shock wave is generated by a "magnetic piston" but no magnetic field exists ahead of it. It is of interest to examine the possibility and conditions of a precursor developing when the plasma ahead of the shock wavefront is magnetized.

The magnetic field may be oriented at various angles to the shock wavefront. Here we confine the discussion to the case when the field in the unperturbed gas is parallel to the shock wavefront and the plane of the thermal layer.

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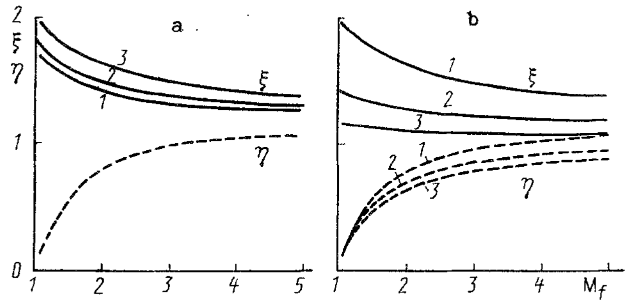

Fig. 1. Curves of ξ and η (solid and dashed curves, respectively) versus M_f during the decay of the shock at the end of the thermal layer. The parameters h (a) and ω (b) are varied.

We assume that the conductivity of the plasma is fairly high and the magnetic field is frozen. We consider the case when a plane shock wave, moving initially in a gas with density ρ_0 and magnetic field H_0 at the end "runs into" a layer of gas with density ρ_T and magnetic field H_T. The layer of reduced density and changed magnetic field has the same thickness Δ along its entire length. For simplicity, the adiabatic exponents γ in the main region in the thermal layer will be considered to be the same. In addition to γ the problem under consideration is also characterized by the following dimensionless parameters: the number M_f of the main shock wave, $\omega = \rho_T/\rho_0$, i.e., the ratio of the density ρ_T in the thermal layer in the unperturbed gas, and $h = H_T/H_0$, i.e., the ratio of the magnetic field strength H_T inside the thermal layer and H_0 outside it, and, finally, $\beta = 8\pi p_0/H_T{}^2$, i.e., the ratio of the initial thermal pressure p_0 to the magnetic "pressure" $H_0{}^2/8\pi$. The Mach number M_f is calculated from the ratio D/c_f of the velocity D_k of the wave to the fast magnetoacoustic velocity of sound $c_f = \sqrt{c_0^2 + H_0^2/4\pi\rho_0}$, where c_0 is the ordinary velocity of sound, i.e., $c_0 = \sqrt{\gamma p_0/\rho_0}$.

The discontinuity decays when the shock wave reaches the end of the thermal layer. In the initial stage the motion is almost uniform. The shock wave propagates along the thermal layer with an initial velocity D_T while a centered rarefaction wave moves in the opposite direction. A qualitative understanding of the distinctive features of such a decay in magnetic gasdynamics can be obtained on the basis that the magnetic field behaves like an impurity of an infinitely light gas with a finite pressure and adiabatic index γ = 2. In the case of a frozen magnetic field the pressure p in the gasdynamics equations is replaced by

$$p_t = p + \frac{H_0^2}{8\pi\rho_0^2}\rho^2,$$

and the internal energy e is replaced by

$$e_t = e + \frac{H_0^2}{8\pi\rho_0^2}\rho.$$

The relation $e_t = p_t/\rho$ holds if γ = 2. All the laws in the case can be obtained from the purely gasdynamic substitutions $p \to p_t$ and $M \to M_f$, while the dependences on the parameters β and h drop out. The effects due to the difference between magnetogasdynamic flow and purely gasdynamic flow are more pronounced when the value of γ differs more from γ = 2.

Let us consider the collapse at various values of ω and h. The field H_0 in the thermal layer (and, hence, the parameter h) can be determined by the process initiating the thermal layer. If the thermal layer is formed fairly rapidly, e.g., because of an "explosion" of the surface layer of the wall or thin foil and expansion of the magnetized plasma of the layer, then as a result of such expansion the amplitude of the frozen field in it decreases in comparison with H_0 and h < 1. If the initiation is rather slow, then the magnetic field manages to diffuse into the plasma of the thermal layer. If the thermal layer came into being as a result of injection of cold gas or evaporation of the wall or foil at a comparatively low phase transition temperature, the thermal layer is not ionized and its formation changes only the density in it and not the amplitude of the magnetic field, whereupon h = 1.

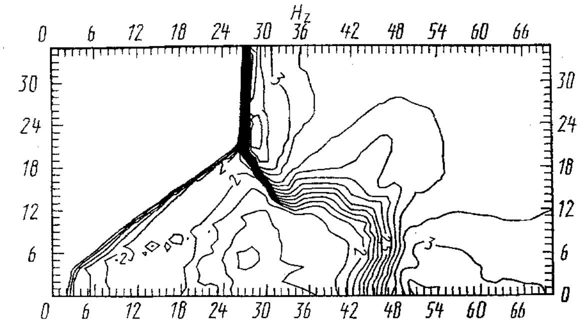

Fig. 2. Lines of equal magnetic field strength at a time when the main shock wave has traveled a distance equal to 19 thermal-layer thicknesses.

Let us give the results of calculation of the decay when $\gamma = 1, 2$. Figure 1a shows the curves of $\xi = D_T/D_0$ versus M_f for $\beta = 1$, $\omega = 0.25$, and various values of h, namely, 0, 0.5, and 1 (curves 1-3, respectively). As $M_f \to 1$ the curves tend to the ratio of the velocity c_f of fast sound in the thermal layer and in the unperturbed gas while as $M_f \to \infty$ they tend to a common limit, which is found from

$$\xi_\infty - 1 = \sqrt{\frac{2}{\gamma - 1}}\,[1 - (\omega\xi_\infty^2)^{\frac{\gamma-1}{\gamma}}]$$

(for $\gamma = 1, 2$, and $\omega = 0.25$ we have $\xi_\infty = 1.37$). The effect of ω on ξ can be traced in Fig. 1b, where curves 1-3 correspond to the values $\omega = 0.25$, 0.5, and 0.75. At h = 1 variation of β from 0.5 to 1.5 (within 5%) does not affect the behavior of the $\xi(M_f)$ curve. The dashed curves in Fig. 1 represent the dependences of the ratio η of the velocity V of contact discontinuity in the thermal layer to the velocity D_0 of the main shock wave.

Under the conditions of a developed and unrestrainedly growing precursor its main part is filled not with the material of the thermal layer but with material of the main part of the gas entering the precursor as a result of vortex motion. In this model of the decay of the discontinuity the thermal layer is filled with material when $V > D_0$, i.e., when $\eta > 1$. As follows from Fig. 1b the condition $\eta > 1$ is realized for the given β and h only when ω decreases to values smaller than some critical value ω_*. The values of h and β play only a small role in this case, as is seen from, e.g., Fig. 1a.

The scheme of the decay of the discontinuity makes for an easy estimate of both the critical value of ω and the role of the magnetic field. The one-dimensional picture, however, is approximate and is valid for times of the order of Δ/d_0, i.e., the time of shock-wave propagation to a distance of the order of the thermal-layer thickness Δ. The motion then becomes essentially two-dimensional and the shock-wave velocity D_T in the thermal layer differs from the initial velocity (immediately after decay). The numerical solution of the corresponding two-dimensional nonstationary magnetogasdynamic problem was used to describe this stage [10].

We give the results of calculation for the case when $\gamma = 1.2$ and $\omega = 0.25$, while $p_0 = H_0^2/8\pi$, i.e., $\beta = 1$. The Alfvén Mach number of the shock wave is $M_A = 3$, i.e., the number

$$M_f = M_A \Big/ \sqrt{\frac{\gamma}{2}\beta + 1} = 2.37.$$

At the beginning of the decay the velocity D_T of shock-wave propagation in the thermal layer is 1.56 times the velocity D_0 of the unperturbed shock wave. The velocity of the contact decay between the material behind the shock wave and the material of the thermal layer is $0.86D_0$, i.e., the boundary of separation lags behind the shock wave. A characteristic precursor structure, in many ways analogous to that in solutions of ordinary gasdynamic problems of a thermal layer, is formed with time [4-7]. Figure 2 shows the lines of equal magnetic field H_z for the time when the shock wave has traversed a distance of 19Δ. The numbers on some of the isolines denote the ratio of the field strength on the line to H_0.

We note that since the initial magnetic field distribution is uniform, the contact decay characteristic of the gasdynamic parameters is expressed indistinctly in the distribution and is due to the difference of the compressions in the shock wave at the tip of the precursor and the oblique shock wave. As $M_f \to \infty$, since the compression in both regions tends to the same value $(\gamma + 1)/(\gamma - 1)$, the decay of the field strength should decrease.

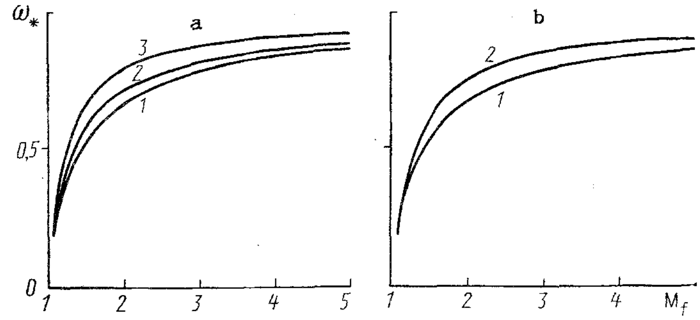

Fig. 3. Dependence of ω_* on M_f when β (a) and h (b) are varied.

The velocity of the precursor tip is $1.27D_0$, i.e., is much lower than the initial velocity of the shock wave in the thermal layer, which naturally is a consequence of the significant role of two-dimensional effects in the "developed" precursor. This velocity is maintained in time and the precursor grows without restraint, the thickness of the thermal layer ceases to play a significant role, and a self-similar stage corresponding to zero thermal-layer thickness is reached. The existence of a thermal-layer effect in a magnetically active plasma has thus been demonstrated theoretically for the first time.

With increasing density ρ in the thermal layer, i.e., with increasing ω, the wave velocity D_T in the precursor decreases and at a critical value $\omega = \omega_*$ becomes equal to D_0 while the thermal layer vanishes. We formulate the criterion for the formation of a precursor by analogy with the criterion in [1], which in fact determines the possibility of a steady-state propagation of a shock wave along a thermal layer. The hypothesis that a large-scale precursor develops if steady-state flow is not possible has not been supported, however, by full-scale computer simulations [2-7]. This is also confirmed by a magnetogasdynamic calculation of flow.

Since the total pressure behind the shock wave in the thermal layer (in the case of steady-state flow the velocity is the same as that of the main shock wave) at $\omega < 1$ is lower than at the pressure p_{1t} behind the main shock wave, according to the Bernoulli equation (generalized to the case of a magnetic field perpendicular to the motion)

$$\frac{v^2}{2} + e + \frac{p}{\rho} + \frac{H^2}{4\pi\rho} = \mathrm{const}$$

the gas of the thermal layer must stagnate in the coordinate system of the main shock wave in order for a precursor to form. Steady-state conditions are possible if the total stagnation pressure of the gas in the thermal layer is higher than p_{1t} and the thermal layer can go to infinity, losing part of its velocity. Otherwise, a steady state is impossible and a large-scale precursor develops. For each h, β, and M_f there exists a critical value ω_* at which the stagnation pressure of the thermal-layer gas is p_{1t}.

Suppose that the "thermal layer" has no magnetic field, i.e., h = 0. In Fig. 3a we show $\omega_*(M_f)$ for $\gamma = 1.2$, $\beta_0 = 0.1$, 1, and 10 (curves 1-3, respectively). All three curves tend to zero as $M_f \to 1$ and to $\omega_\infty = \left(1 + \frac{(\gamma-1)^2}{4\gamma}\right)^{\frac{\gamma}{\gamma-1}}$ as $M_f \to \infty$. At intermediate values of the numbers M_f the magnetic field suppresses the formation of a precursor, first because the effective adiabatic exponent grows and, second, because the sound velocity rises and the Mach number decreases. When γ increases the difference between the curves at various β becomes smaller and smaller and vanishes altogether at $\gamma = 2$. The criterion at zero magnetic field in the thermal layer is shown in Fig. 3b for $\beta = 0.1$ and $\gamma = 1, 2$. Curves 1 and 2 correspond to h = 0.1 and 1. As in the case when β is varied as γ increases the difference between the dependences at different h decreases. For $\gamma = 2$ the critical value ω_* does not depend on β and h, and we have

$$\omega_* = \frac{2}{M_f^2}\left[\sqrt{\frac{1}{3}(4M_f^2 - 1)} - 1\right] \quad \text{for} \quad \omega_* M_f^2 \leqslant 1,$$

$$\omega_* = \frac{32 M_f^2 - 8 + 2\sqrt{(4 M_f^2 - 1)(64 M_f^2 - 97)}}{81 M_f^2} \quad \text{for} \quad \omega_* M_f^2 > 1.$$

The situation when the magnetic field is not parallel to the shock wavefront is much more complicated because anisotropy characteristic of magnetogasdynamic processes appears and additional analysis is required.

In conclusion we note that the thermal-layer effect in a magnetically active plasma not only determines the development of wall disturbances in a MGD tube, Z pinches, and plasma focuses, but may also be significant in the absence of walls, i.e., in geophysical and astrophysical problems, when a channel of reduced density is the trace of a fast meteoroid, asteroid, or planet.

NOTATION

γ, adiabatic exponent; ρ, p, e, H, v, density, gasdynamic pressure, internal energy, magnetic field strength, and velocity; β, ratio of gasdynamic pressure to magnetic pressure; D_0, D_T, velocity of the main shock wave and the shock wave in the thermal layer; ω, ratio of ρ_T to ρ_0; ω_*, critical value of ω; h, ratio of H_T to H_0; V, velocity of contact discontinuity; η, ratio of D_T and V to D_0; M_A, Alfvén Mach number; c_f, fast magnetogasdynamic velocity of sound; M_f, Mach number calculated from c_f; Δ, thickness of the thermal layer. Subscripts: 0, parameters ahead of the front of the main shock wave; T, parameters ahead of the front of the shock wave in the thermal layer.

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